Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 17 January 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

H26315RBA

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

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2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n + 1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

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3. $$\mathrm{f}(x) = \ln(x + 2) - x + 1, \qquad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $\mathrm{f}(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $\mathrm{f}(x) = 0$ correct to 3 decimal places.

(2)

4.

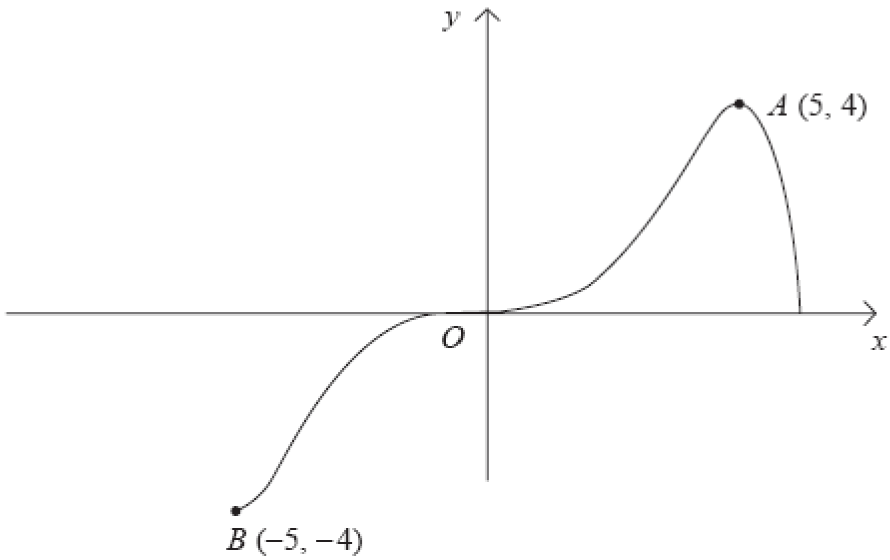

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.

The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}(|x|)$,

(3)

(*c*) $y = 2\mathrm{f}(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to *A* and *B*.

5. The radioactive decay of a substance is given by

$$R = 1000\mathrm{e}^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(*a*) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(*b*) Find the value of c to 3 significant figures.

(4)

(*c*) Calculate the number of atoms that will be left when $t = 22\,920$.

(2)

(*d*) Sketch the graph of R against t .

(2)

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6. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(*b*) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

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7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \qquad -\pi \le x \le \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places.

(4)

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8. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}.$$

$$\mathrm{g} : x \mapsto \frac{3}{x} - 4, \quad x > 0,\ x \in \mathbb{R}.$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$\mathrm{gf} : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $\mathrm{gf}(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = \mathrm{gf}(x)$.

(5)

TOTAL FOR PAPER: 75 MARKS

END

edexcel

January 2008
6665 Core C3
Mark Scheme

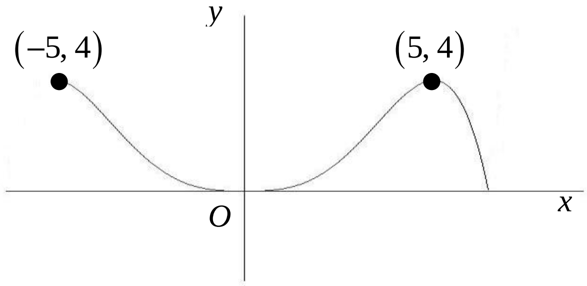
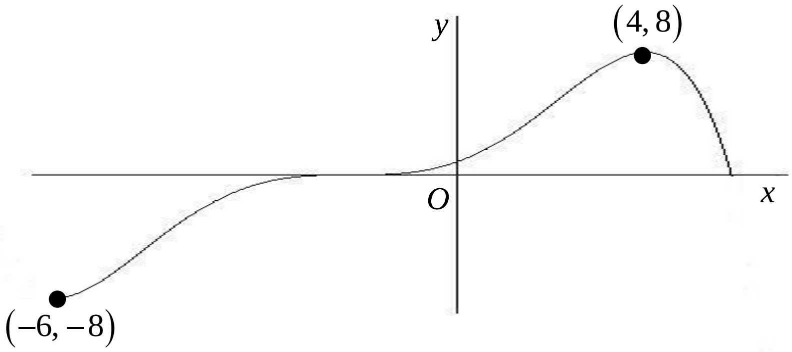
Final Version

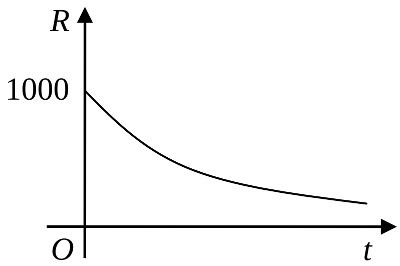
<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1.</td><td>
$$\begin{array}{r|l} & \quad 2x^2 \qquad -1 \\ \hline x^2-1 & 2x^4 \quad -3x^2+x+1 \\ & 2x^4 \quad -2x^2 \\ \hline & \qquad -x^2+x+1 \\ & \qquad -x^2 \qquad +1 \\ \hline & \qquad\qquad x \end{array}$$
$a=2$ stated or implied
$c=-1$ stated or implied
$$2x^2-1+\frac{x}{x^2-1}$$
$a=2,\ b=0,\ c=-1,\ d=1,\ e=0$
$d=1$ and $b=0,\ e=0$ stated or implied
</td><td>M1
A1
A1

A1
[4]</td></tr>
<tr><td>2.</td><td>
(a)
$$\frac{dy}{dx}=2e^{2x}\tan x+e^{2x}\sec^2 x$$
$$\frac{dy}{dx}=0 \quad\Rightarrow\quad 2e^{2x}\tan x+e^{2x}\sec^2 x=0$$
$$2\tan x+1+\tan^2 x=0$$
$$(\tan x+1)^2=0$$
$$\tan x=-1 \quad *$$
cso
(b)
$$\left(\frac{dy}{dx}\right)_0=1$$
Equation of tangent at $(0,0)$ is $y=x$
</td><td>M1 A1+A1
M1
A1
A1 (6)

M1
A1 (2)
[8]</td></tr>
</table>

Question Number	Scheme		Marks	
3.	(a) $\mathrm{f}(2) = 0.38 \ \ldots$			
	$\mathrm{f}(3) = -0.39 \ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ ∗	cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$x_2 \approx 2.50498$		A1	
	$x_3 \approx 2.50518$		A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$\mathrm{f}(2.5045) \approx 6 \times 10^{-4}$			
	$\mathrm{f}(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$			
	$\Rightarrow$ root $= 2.505$ to 3 dp ∗	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

Question Number	Scheme	Marks	
4.	(a)		
			
	Shape	B1	
	$(5, 4)$	B1	
	$(-5, 4)$	B1	(3)
	(b) For the purpose of marking this paper, the graph is identical to (a)		
	Shape	B1	
	$(5, 4)$	B1	
	$(-5, 4)$	B1	(3)
	(c)		
			
	General shape – unchanged	B1	
	Translation to left	B1	
	$(4, 8)$	B1	
	$(-6, -8)$	B1	(4)
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.		**[10]**

Question Number	Scheme		Marks	
5.	(a) 1000		B1	(1)
	(b) $1000\,e^{-5730c} = 500$		M1	
	$e^{-5730c} = \frac{1}{2}$		A1	
	$-5730c = \ln\frac{1}{2}$		M1	
	$c = 0.000121$	cao	A1	(4)
	(c) $R = 1000\,e^{-22920c} = 62.5$	Accept 62-63	M1 A1	(2)
	(d) 	Shape 1000	B1 B1	 (2) **[9]**

Question Number	Scheme	Marks
6.	(a) $\cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$	M1
	$= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$	M1
	$= (2\cos^2 x - 1)\cos x - 2(1-\cos^2 x)\cos x$ any correct expression	A1
	$= 4\cos^3 x - 3\cos x$	A1 (4)
	(b)(i) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$	M1
	$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$	A1
	$= \dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$	M1
	$= \dfrac{2}{\cos x} = 2\sec x$ ✱ cso	A1 (4)
	(c) $\sec x = 2$ *or* $\cos x = \dfrac{1}{2}$	M1
	$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ accept awrt 1.05, 5.24	A1, A1 (3)
		[11]
7.	(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6\cos 2x - 8\sin 2x$	M1 A1
	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 6$	B1
	$y - 4 = -\dfrac{1}{6}x$ or equivalent	M1 A1 (5)
	(b) $R = \sqrt{(3^2+4^2)} = 5$	M1 A1
	$\tan\alpha = \dfrac{4}{3}$, $\alpha \approx 0.927$ awrt 0.927	M1 A1 (4)
	(c) $\sin(2x + \text{ their } \alpha) = 0$	M1
	$x = -2.03, -0.46, 1.11, 2.68$	A1 A1 A1 (4)
	First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the *y*-coordinate.	**[13]**

Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3 - 4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3 - 1}{1-2x^3}$ * cso	A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find *y*.	M1
	$x = 0, y = -1$	A1 (5) **[13]**